HYPERPLANE ARRANGEMENTS

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The theory of hyperplane arrangements is a very active research topic, with many interesting open questions to attack and deep relations with several branches of Mathematics, such as Combinatorics, Algebraic Topology and Algebraic Geometry.

In our M2 lectures we will cover the following subjects.

1. The combinatorics of hyperplane arrangements

Given an arrangement, that is a finite collection of hyperplanes $\mathcal{A}$ in a finite dimensional vector space V, we will introduce the associated intersection lattice $L(\mathcal{A})$ and various characteristic polynomials associated with the arrangement $\mathcal{A}$ which are computed using linear algebra and the Möbius function of the lattice $L(\mathcal{A})$. This part will follow the references [OT, Chap.2], [S, Lectures 1 and 2] and [D2, Chap.2], and it requires practically no prerequisites.

2. Cohomology of hyperplane arrangement complements

From now on we work over the complex number field $\mathbb{C}$ and give the combinatorial description of the cohomology $H^*(M(\mathcal{A}))$ of the complement $M(\mathcal{A}) = V \setminus \cup_{H\in\mathcal{A}} H$ in terms of the Orlik-Solomon algebra of the arrangements. This part will follow essentially the references [OT, Chap. 3 and 5] and [D2, Chap. 3], and it requires some familiarity with (singular or de Rham) cohomology, as for instance in the book [BT] .

3. Milnor fibers and local system cohomology

In this part we will discuss the cohomology $H^*(F(\mathcal{A}))$ of the Milnor fiber $F(\mathcal{A})$ associated to a hyperplane arrangement. In fact, there is a monodromy action on $H^*(F(\mathcal{A}))$, and the corresponding eigenspaces can be expressed using the cohomology of the complement $M(\mathcal{A})$ with coefficients in rank one local systems. This part will follow essentially the reference [D1, Chap. 6] or [D2, Chap. 5-6], and it requires at least some familiarity with twisted cohomology, as for instance in the books [BT], [D2]. A lot of examples will be provided along the way, and the exercises in the book [D2] will form a basis for our tutorials.

References

[BT] Bott, R., Tu, L.W.: *Differential Forms in Algebraic Topology.* Graduate Texts in Maths. **82**, Springer, Berlin Heidelberg New York (1982).

[D1] Dimca, A.: *Sheaves in Topology*, Universitext, Springer, 2004.

[D2] Dimca, A.: *Hyperplane Arrangements: An Introduction*, Universitext, Springer, 2017.

[OT] Orlik, P., Terao, H.: *Arrangements of Hyperplanes.* Grundlehren Math. Wiss., **300**, Springer-Verlag, Berlin (1992).

[S] Stanley, R. : *An Introduction to Hyperplane Arrangements,* Park City Mathematics Series, volume 14: Geometric Combinatorics (2004).