Finite volume schemes for diffusion equations

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There are many different discretization strategies to handle PDEs: Finite Differences, Finite Elements, Finite Volumes, Spectral methods... The purpose of this lecture is to present Finite Volume methods for diffusion equations on generic meshes. Finite Volume are intended to mimic the relations obtained by integrating over cells the PDE under consideration, and they reproduce at the discrete level the balance principles that have led to the derivation of the equations. Such methods apply to many different fields: fluid mechanics, heat and mass transfer in heterogeneous media,...

After introducing the main ideas and construction principles of the methods, we review some results of the recent literature, focusing on two important properties of schemes (discrete versions of well-known properties of the continuous equation): coercivity and minimum-maximum principles. Coercivity ensures the stability of the method as well as its convergence under assumptions compatible with real-world applications, whereas minimum-maximum principles are crucial in case of strong anisotropy to obtain physically meaningful approximate solutions. The outline of the lecture is the following:

- Presentation of the mains ideas of the finite volume methods.
- Study in dimension 1:
  - Coercivity, wellposedness and minimum-maximum principles of the discrete solution.
  - Convergence of the discrete solution.
- Extension to dimension 2:
  - Study of the two points flux approximation (classical strategy).
  - Study of a more general method called Discrete Duality Finite Volume.
- Numerical simulations using scilab.